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BI - AND QUASI-IDEAL SEMIGROUPS WITH n-PROPERTY

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We give a structure description for each semigroup belonging to the classes in the title which we define in a similar way as it was done in /3/ for left-ideal semigroups with n-property.

1. SOME PRELIMINARY RESULTS

Let S be a semigroup. We shall denote by E_S the set of idempotents of S.

THEOREM 1. *A semigroup S is periodic and the mapping $\varphi: S \rightarrow E_S$, defined by $\varphi(x)=e_x$ where e_x is the idempotent in $\langle x \rangle$, is a homomorphism iff for every $a,b \in S$, $n \in N$ there exists $r \in N$ such that $(ab)^r=(a^nb^n)^r$ and $E_S^2=E_S$.*

Proof. Let S be a periodic semigroup and φ a homomorphism, where φ is defined as above. Then $\ker\varphi$ is a congruence with the congruence classes

$$K_e=\{x \in S \mid (n \in N)\ x^n=e\}, \quad e \in E$$

which are power joined semigroups. Hence, according to Theorem 1 /7/, it follows that for every $a,b \in S$, $n \in N$ there exists $r \in N$ such that $(ab)^r=(a^nb^n)^r$. Furthermore since φ is an epimorphism, we have that $S/\ker\varphi = E_S$ which implies that $E_S^2=E_S$.

Conversely, for every $a,b \in S$, $n \in N$ let there be on $r \in N$ such that $(ab)^r=(a^nb^n)^r$; then $a^{2r}=a^{2nr}$ and S is periodic. If we put

$$a\rho b \overset{def}{\Longleftrightarrow} (\exists n \in N)\ a^n=b^n$$

then ρ will turn out to be a band congruence and the congruence classes mod ρ will be periodic unipotent power joined semigroups (/7/ Theorem 1), and then the mapping φ defined by $\varphi(x)=e_x$ will be an epimorphism from S onto E_S.

COROLLARY 1. *A semigroup S is periodic, E_S a rectangular band and $\varphi: S \rightarrow E_S$ $(\varphi(x)=e_x)$ a homomorphism iff for every $a,b,c \in S$, $n \in N$ there exists an $r \in N$ such that $(abc)^r=(ac)^{nr}$, $E_S^2=E_S$.*

Proof. Follows from (/7/ Theorem 3) and (/7/,Theorem 1).

Let S be a semigroup with zero 0; we call S a nil-semigroup iff for every $a\in S$ there is an $n\in N$ such that $a^n=0$.

LEMMA 1. _A semigroup S is a nil-semigroup iff for every a,b∈S there is an n∈N such that_ $a^n=b^{n+1}$.

Proof. If S is a nil-semigroup then the statement in the Lemma 1 is obvious.

Conversely, for every $a,b\in S$ let there be an $n\in N$ such that $a^n=b^{n+1}$. Then for a=b we have that $a^n=a^{n+1}$ which implies that a^n is an idempotent; furthermore, from $a^n=a^{n+1}$ it follows that a^n is the zero in $\langle a\rangle$. Let us show that a^n is zero in S; let $b\in S$ is an arbitrary element. From the above discussion it follows that for same $k\in N$, b^k is zero in $\langle b\rangle$. Now, there exists an $m\in N$ such that $(a^n)^m=(b^k)^{m+1}$ and, since a^n, b^k are idempotents we have that $a^n=b^k$ which means that a^n is zero for b. So, S has a zero and is a nil-semigroup.

THEOREM 2. _A semigroup S is a band of nil-semigroups iff the following properties are satisfied:_

1. $(\forall x\in S)(\exists r\in N)\ x^r=x^{r+1}$,
2. $(\forall x,y\in S)\ (\forall n\in N)(\exists r\in N)\ (xy)^r=(x^ny^n)^r$.

Proof. Let S be a band Y of nil-semigroups S_α, $\alpha\in Y$. Then according to Lemma 1 we have that 1 is satisfied. Since every nil-semigroup is a power joined semigroup, if follows that 2 is satisfied too /7, Th.1/.

Conversely, let the conditions 1 and 2 be satisfied. Then S will be a band Y of periodic power joined semigroups S_α, $\alpha\in Y$ /7, Th.1/. So, for $a,b\in S_\alpha$, $\alpha\in Y$, we have that $a^n=b^n$, $b^k=b^{k+1}$ for some $n,k\in N$ and,

$$a^{nk}=b^{nk}=b^{nk-k}b^k = b^{nk+1}$$

which, according to Lemma 1, implies that S_α is a nil-semigroup.

Let E be a band, P a partial semigroup, $E\cap P=\emptyset$, and $\varphi:P\to E$ a partial homomorphism. Let us extend φ to a mapping $\psi:S=E\cup P\to E$ by $\psi(x)=\varphi(x)$ if $x\in P$ and $\psi(e)=e$ for all $e\in E$. Let us define an operation on S by

$$xy = \begin{cases} xy \text{ as in } P, \text{if } x,y\in P \text{ and } xy \text{ is defined in } P \\ \psi(x)\psi(y), \text{otherwise} \end{cases}$$

Then S will become a semigroup with E an ideal and ψ an epimorphism. In what follows we shall denote the semigroup S constructed above by $S=(E,P,\varphi)$.

A partial semigroup P is said to be a power breaking partial semigroup iff for every $x\in P$ there exists a $k\in N$ such that x^k is not defined in P.

THEOREM 3. _The following conditions on a semigroup S are equivalent:_

(i) _S is periodic,_ $\varphi:S\to E_S$ $(\varphi(x)=e_x)$ _is a homomorphism and_ $(\forall x\in S)$ $(\forall e\in E_S)$ $xe, ex\in E_S$;

(ii) $(\forall a,b\in S)(\forall n\in N)(\exists r\in N)\ (ab)^r=(a^nb^n)^r$ and $(\forall x\in S)(\forall e\in E)\ xe,ex\in E_S$;

(iii) $S\cong(E,P,\varphi)$ where P is a power breaking partial semigroup.

Proof. From Theorem 1 it follows that (i) ⇒ (ii). If (ii) is true, from the proof of Theorem 1 it follows that S is periodic and, since $xe,ex\in E_S$ for every $x\in S$, $e\in E_S$, we have that E_S is an ideal in S. So, if we put $P=S\setminus E$, we will have that P is a partial power breaking semigroup. According to Theorem 1, the mapping $\varphi|_P(\varphi(x)=e_x)$ will be a partial homomorphism from P to E_S suchthat $\varphi(e)$ = for all $e\in E_S$. So, we have that $S\cong(E,P,\varphi)$ and we have proved that (ii) ⇒ (iii). It is obvious that (iii) ⇒ (i).

2. BI-IDEAL SEMIGROUPS WITH n-PROPERTIES

A subsemigroup B of a semigroup S is said to be a bi-ideal iff $BSB\subseteq B$. The principal bi-ideal $B[a]$ of a semigroup S generated by $a\in S$ is $B[a]=a\cup a^2\cup aSa$.

A semigroup S is said to be a c-bi-ideal semigroup iff every cyclic subsemigroup $\langle a\rangle$ of S is a bi-ideal of S.

THEOREM 4. The following conditions on a semigroup S are equivalent:

(i) S is a c-bi-ideal semigroup;

(ii) $(\forall a\in S)\ aSa\subseteq\langle a\rangle$;

(iii) $(\forall a\in S)\ \ B[a]=\langle a\rangle$.

Proof. From $aSa\subseteq\langle a\rangle S\langle a\rangle\subseteq\langle a\rangle$ it follows that (i) ⇒ (ii). It is obvious that (ii) ⇒ (iii). Let (iii) be satisfied and let $\langle b\rangle$ be a cyclic subsemigroup of S. Then for $b^i,b^j\in\langle b\rangle$ we have that

$$b^iSb^j=b^{i-1}bSbb^{j-1}\subseteq b^{i-1}B\,b\,b^{j-1}=$$
$$=b^{i-1}\langle b\rangle b^{j-1}\subseteq\langle b\rangle,$$

and so, $\langle b\rangle S\langle b\rangle\subseteq\langle b\rangle$ which means that S is a c-bi-ideal semigroup.

Let us recall that S is a bi-ideal semigroup iff every subsemigroup of S is a bi-ideal in S ([2]) and that the bi-ideal $B[C]$ generated by the non-empty subset C of the semigroup S is $B[C]=C\cup C^2\cup CSC$. In a simmilar way as in the case of Theorem 4, the following can be proved:

THEOREM 5. The following conditions on a semigroup S are equivalent:

(i) S is a bi-ideal semigroup;

(ii) $CSC\subseteq\langle C\rangle$, for every non-empty subset C of S;

(iii) $B[C]\subseteq\langle C\rangle$.

A partial subsemigroup R of a partial semigroup P is a bi-ideal in P iff r_1pr_2 is defined in P, $r_1,r_2\in R$, $p\in P$ implies $r_1pr_2\in R$. If every partial subsemigroup of a partial semigroup P is a bi-ideal in P, we call P a partial bi-ideal semigroup.

THEOREM 6 [2]. A semigroup S is a bi-ideal semigroup iff $S \cong (E,P,\varphi)$ where E is a rectangular band and P a partial power breaking bi-ideal semigroup.

We call partial semigroup P a partial c-bi-ideal semigroup iff whenever apa is defined in P, apa $\in \langle a \rangle$ where $\langle a \rangle$ consists of all powers a^n which are defined in P. In a similar way as Theorem 6, the following can be proved:

THEOREM 7. A semigroup S is a c-bi-ideal semigroup iff $S \cong (E,P,\varphi)$ where E is a rectangular band and P a partial power breaking c-bi-ideal semigroup.

It is obvious that the class of c-bi-ideal semigroups is more general than the class of bi-ideal semigroup.

Let S be a semigroup and Q a subset of S. We call S a

(i) β_0^n - semigroup iff $Q \subseteq S$, $Q^{n+1} \subseteq Q \to QSQ \subseteq Q$;

(ii) β_1^n - semigroup iff $Q \subseteq S$, $Q^{n+1} \subseteq Q \to QS^{n-1}Q \subseteq Q$;

(iii) β_2^n - semigroup iff $Q \subseteq S$, $Q^2 \subseteq Q \to QS^{n-1}Q \subseteq Q$.

Observe that for n=1 β_0^n -, β_2^n - semigroups are simply bi-ideal semigroups. It is easily seen that:

LEMMA 2. Every subsemigroup and every homomorphic image of a β_0^n-, β_1^n - semigroup is also a β_0^n-, β_1^n-, β_2^n-semigroup, respectively.

LEMMA 3. (i) Every β_0^n-semigroup is a β - semigroup;

(ii) every β - semigroup is a β_2^n-semigroup;

(iii) every β_1^n-semigroupis a β_2^n - semigroup,

where β- semigroup stands for bi-ideal semigroup.

LEMMA 4. Let S be a semigroup. If S is a β-β_0^n-semigroup then $aSa \subseteq \langle a \rangle$ for every $a \in S$; if S is a β_1^n-, β_2^n-semigroup then $a S^{n-1} a \subseteq \langle a \rangle$ for every $a \in S$.

LEMMA 5. Let S be a β_0^n-, β_1^n-semigroup. Then:

(i) S is periodic; and for every $a \in S$ the periodic part H_a of $\langle a \rangle$ is a trivial subgroup of S;

(ii) E is a rectangular band which is an ideal in S; for every $e \in E$, $x \in S$, exe=e;

(iii) if xyx=x for some $y \in S$, then $x \in E$.

LEMMA 6. (i) If S is a β-semigroup, then $|\langle a \rangle| \leqslant 5$ for every $a \in S$;

(ii) if S is a β_0^n-semigroup, then $|\langle a \rangle| \leqslant 3$ for every $a \in S$;

(iii) if S is a β_1^n-, β_2^n-semigroup, then $|\langle a \rangle| \leqslant n+3$ for every a S.

Proof. Let, for example, S be a β_1^n-semigroup, $a \in S$ and let $\langle a \rangle_n = \{a, a^{n+1}, a^{2n+1}, \ldots\}$ be the n-subsemigroup of S generated by a. Since

$$a^{n+2} = a \cdot a^2 \cdot a^{n-2} \cdot a \in \langle a \rangle_n S^{n-1} \langle a \rangle_n \subseteq \langle a \rangle_n,$$

we have that $a^{n+2}=a^{kn+1}$ for some $k\in N$, which means that the index r_a of a is $\leqslant n+2$ and, since the periodic part of $\langle a\rangle$ consists of one element (Lemma 5 (i)), we have that $|\langle a\rangle|\leqslant n+3$.

Let P be a partial semigroup. Then P is said to be a: (i) β_0^n-semigroup iff for every $Q\subseteq P$ which posseses the property $q_0q_1\dots q_nq\in Q$, $q_j\in Q$ whenewer $q_0q_1\dots q_n$ is defined in P we have that, if $q_1^*pq_2^*$ is defined in P, then $q_1^*pq_2^*\in Q$, $q_1^*,q_2^*\in Q$, $p\in P$; (ii) β_1^n-semigroup iff for every $Q\subseteq P$ which possesses the property mentioned in (i), $q_1^*p_1p_2\dots p_{n-1}q_2^*$ defined in P implies $q_1^*p_1p_2\dots p_{n-1}q_2^*\in Q$; (iii) β_2^n-semigroup iff for every $Q\subseteq P$ such that whenever q_1q_2 is defined in P, if $q_1q_2\in Q$ then the following is true: if $q_1^*p_1p_2\dots p_{n-1}q_2^*$ is defined in P then $q_1^*p_1p_2\dots p_{n-1}q_2^*\in Q$, $q_1^*,q_2^*\in Q$, $p_j\in P$.

THEOREM 8. _A semigroup S is a β_0^n-semigroup iff $S\cong(P,E,\varphi)$ where E is a rectangular band and P a partial power breaking β_0^n-semigroup._

Proof. Let S be a β_0^n-semigroup. From Lemma 5 it follows that $S=E\cup P$ ($P=S\setminus E$) where E is a rectangular band and ideal in S, and P is a power breaking partial semigroup. Let $Q\subseteq P$ possess the property $q_0q_1\dots q_n\in Q$ whenever $q_0q_1\dots q_n$ is defined in P, $q_i\in Q$ and let $Q^*=Q\cup E$. Then $Q^{*n+1}\subseteq Q^*$; since S is a β_0^n-semigroup, it follows that $Q^*SQ^*\subseteq Q^*$. If $q_1^*,q_2^*\in Q$, $q_1^*pq_2^*\notin E$, we conclude that $q_1^*pq_2^*\in Q$ which proves that P is a partial β_0^n-semigroup. Finally, if we put $\varphi(x)=e_x$, e_x is the idempotent in $\langle x\rangle$, we can easily show that $\varphi:P\to E$ is a homomorphism (as in [2] and by Theorem 3 we have that $S\cong(E,P,\varphi)$.

Conversely, let $S\cong(E,P,\varphi)=T$ with E,P as stated in the Theorem and let $B\subseteq T$, $B^{n+1}\subseteq B$. Then $B^*=B\setminus E$ posseses the property $b_0b_1\dots b_n\in B^*$ whenever $b_0b_1\dots b_n$ is defined in P, so, if $b,c\in B^*$, $p\in P$, then $bpc\in B^*\subseteq B$. Let for $b,c\in B$, $t\in T$, $btc\in E$. If $bc\notin B^*$, then b.c is not defined in P and,

$$btc=\varphi(b)\varphi(t)\varphi(c)=\varphi(b)\varphi(c)=[\varphi(b)]^n\varphi(c)=\varphi(b^n)\varphi(c)=b^nc\in B.$$

If $bc\in P$ then $(bc)^k\in E$ since P is a power breaking partial semigroup. Let $(bc)^k=e$, then

$$btc=\varphi(b)\varphi(t)\varphi(c)=\varphi(b)\varphi(c)=\varphi(bc)=\varphi[(bc)^k]=e.$$

On the other hand we have that

$$e=\varphi(b)\varphi)c)=\varphi(b^{kn}c)=b^{kn}c\in B^{kn+1}\subseteq B$$

if $b^{kn}\in E$. Now, from $bc\in P$ it follows that $b\in P$ and there exists an $m\in N$ such that b^m is not defined in P; then for $k\in N$, $kn\geqslant m$ we have that $b^{kn}\in E$ since E is an ideal in T. So, we have proved that $btc\in B$ for every $b,c\in B$, $t\in T$, which completes the proof.

In a similar way the following can be proved:

THEOREM 9. *A semigroup S is a β_1^n-semigroup iff $S \cong (E,P,\varphi)$ where E is a rectangular band, P a partial power breaking β_1^n-semigroup.*

THEOREM 10. *A semigroup S is a β_2^n-semigroup iff $S \cong (E,P,\varphi)$ where E is a rectangular band and P a partial power breaking β_2^n-semigroup.*

3. QUASIIDEAL SEMIGROUPS WITH n-PROPERTY

In a similar way as in part 2 we can introduce the following classes of semigroups: We call a semigroup S:

(i) q_0^n-semigroup iff $Q \subseteq S$, $Q^{n+1} \subseteq Q \Rightarrow QS \cap SQ \subseteq Q$;

(ii) q-semigroup iff $Q \subseteq S$, $Q^2 \subseteq Q \Rightarrow QS \cap QS \subseteq Q$;

(iii) q_1^n-semigroup iff $Q \subseteq S$, $Q^{n+1} \subseteq Q \Rightarrow QS^n \cap S^nQ \subseteq Q$;

(iv) q_2^n-semigroup iff $Q \subseteq S$, $Q^2 \subseteq Q \Rightarrow QS^n \cap S^nQ \subseteq Q$

We are not going to reformulate all the results for the semigroups defined above; these results are similar to those in 2. We shall do this only for some of these semigroups, including the theorems which give a structure description for each of these semigroups.

LEMMA 7. *Let S be a semigroup.*

(i) *If S is a q-semigroup, then* $|\langle a\rangle| \leqslant 3$ *for every* $a \in S$;

(ii) *if S is a q_0^n-semigroup, then* $|\langle a\rangle| \leqslant 2$ *for every* $a \in S$;

(iii) *if S is a q_1^n-, q_2^n-semigroup, then* $|\langle a\rangle| \leqslant n+2$ *for every* $a \in S$.

THEOREM 11. *A semigroup S is a q_0^n-semigroup iff $S \cong (E,P,\varphi)$ where E is a rectangular band, P a nonempty set and $\varphi: P \to E$ a mapping.*

Proof. Since every quasiideal of a semigroup is a bi-ideal tool, it follows that a q_0^n-semigroup is a β_0^n-semigroup too. So, we can use all the properties which a β_0^n-semigroup possesses. Let S be a q_0^n-semigroup, $x,y \in S$ and let $Q=\{x,y,e_x,e_y,e_x e_y,e_y e_x\}$. We shall show that $Q^{n+1} \subseteq Q$. For n=2 we have the following possibilities: (i) if in q=abc all of a,b and c are idempotents, then (E is a rectangular band and ex, xe $\in E$ by Lemma 5) $q=e_xe_y$, e_ye_x, e_x, e_y; $q=ac=e_xe_y$ if $a=e_x$, $c=e_y$. (ii) if one of a,b and c is idempotent, for example if b is idempotent, then ab and bc will be idempotents also and q=abc=a.bcb= ab.cbc=abe=abab.e_c=e_abe_c=e_ae_c and again $q \in Q$; (iii) since $x^2=e_x$, $y^2=e_y$ (Lemma 7 (ii)), the product q=abc doesn't contain any idempotent in the following two cases: $q=xyx=e_x$ and $q=yxy=e_y$ since from $xyx \in \langle x\rangle$ if follows that xyx=x (and then x is idempotent) if $xyx=e_x$ and similar for yxy. So, if n=2 we have proved that $Q^{n+1} \subseteq Q$. Now, let n>2; then according to previous considerations, in any product of n+1 elements from Q, the product of any three elements, as we have shown above, will be equal to an idempotent and, accordingly, in a similar way we can prove that all the product is equal to an idempotent which belongs to Q. So, Q will be an n-subsemigroup of S, i.e. $Q^{n+1} \subseteq Q$. From this it follows that Q is a quasiideal in S which implies that Q is a subsemigroup of S; we have proved that $xy \in Q$. If $xy=y$ then $y=xy=x^2y=e_xy=e_xy=e_xe_y$ which is impossible if we take x and y not to be idempotents. Similarly for $xy=x$. So, xy must be an idempotent:

$$xy=xyx.y = e_xy=e_x.ye_xy = e_xe_y$$

Now, if we put $P=S\setminus E$, we have that for every $x,y \in P$, $xy \in E$. Furthermore, with $\varphi(x)=e_x$, $x \in P$ and e_x the idempotent in $\langle x \rangle$ we can define a mapping from P to E which can be considered as a partial homomorphism from P to E and Theorem 3 concludes the proof.

Conversely, let $Q \subseteq T = E \cup P$ where E is a rectangular band, P a set such that $E \cap P = \emptyset$ and let $\varphi: P \to E$ be a mapping, and let $Q^{n+1} \subseteq Q$. If $x \in QT \cap TQ$, i.e. $x=q_1x_1=x_2q_2$, $q_1,q_2 \in Q$, $x_1,x_2 \in T$, we have that $x \in E$ and, according to the definition of operation in (E,P,φ), we have that

$$x=x^2=q_2x_2x_1q_1=\varphi(q_2)\varphi(x_2)\varphi(x_1)\varphi(q_1)=\varphi(q_2)\varphi(q_1)=\varphi(q_2^n)\varphi(q_1) = =\varphi(q_2^nq_1) = q_2^nq_1 \in Q$$

which shows that Q is a quasi-ideal in T.

Let P be a partial semigroup. Then P is said to be a: (i) q-semigroup iff for every $Q \subseteq P$ which posseses the property $q_1q_2 \in Q$ whenever q_1q_2 is defined in P, $q_1,q_2 \in Q$, we have that, if pq is defined in P, $p \in P$, $q \in Q$ and, for some $p' \in P$, $q' \in Q$ $pq=q'p'=x$, then $x \in Q$; (ii) q_1^n-semigroup iff for every $Q \subseteq P$ which posseses the property $q_0q_1\ldots q_n \in Q$ whenever $q_0q_1\ldots q_n$ is defined in P, $q_i \in Q$ we have that, if $p_1p_2\ldots p_nq$ is defined in P for $p_i \in P$, $q \in Q$ and for some $p_i' \in P$, $q' \in Q$, $p_1p_2\ldots p_nq=q'p_1'p_2'\ldots p_n' = x$ then $x \in Q$; (iii) q_2^n-semigroup iff for every $Q \subseteq P$ which posseses the property $q_1q_2 \in Q$, $q_1,q_2 \in Q$, whenever q_1q_2 is defined in P we have that if $p_1p_2\ldots p_nq$ is defined in P, $p_i \in P$, $q \in Q$ and for some $p_i' \in P$, $q' \in Q$, $p_1p_2\ldots p_nq=q'p_1'p_2'\ldots p_n' = x$ then $x \in Q$.

Using a similar procedure as for Theorem 8, and using also Theorem 9 and 10, the following can be proved:

THEOREM 12. _A semigroup S is a q-semigroup iff_ $S \cong (E,P,\varphi)$ _where E is a rectangular band and P a partial power breaking q-semigroup._

THEOREM 13. _A semigroup S is a_ q_1^n_-semigroup iff_ $S \cong (E,P,\varphi)$ _where E is a rectangular band and P a partial power breaking_ q_1^n_-semigroup._

THEOREM 14. A semigroup S is a q_2^n-semigroup iff S $\cong$ (E,P,φ) where E is a rectangular band and P a partial power breaking q_2^n-semigroup.

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